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Passive Dynamic Controllers for Nonlinear Mechanical Systems

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A methodology for model-independent controller design for controlling the large angular motion of multibody dynamic systems is outlined. The controlled system may consist of rigid and flexible components that undergo large rigid body motion and small elastic deformations. Control forces/torques are applied to drive the system and at the same time suppress the vibrations due to flexibility of the components. The proposed controller consists of passive second-order systems that may be designed with little knowledge of the system parameters, even if the controlled system is nonlinear. Under rather general assumptions, the passive design assures that the closed-loop system has guaranteed stability properties. Unlike positive real controller design, stabilization can be accomplished without direct velocity feedback. In addition, the second-order passive design allows dynamic feedback controllers with considerable freedom to tune for desired system response and to avoid actuator saturation. After developing the basic mathematical formulation of the design methodology, simulation results are presented to illustrate the proposed approach applied to a flexible six-degree-of-freedom manipulator.

Introduction

IN this paper, a controller design methodology is outlined for nonlinear dynamic systems based on simulating the force/torque histories that would be applied by a chosen virtual system of masses, springs, and dashpots. In a companion paper by Juang and Phan,[1] the corresponding theory for linear dynamic systems is presented. Here, nonlinear problems are addressed by generalizing the standard Lyapunov stability theory to allow less restrictive conditions on the Lyapunov function. A major advantage of the control approach is its ability to guarantee stability of the controlled system with very little knowledge of the system being controlled. It has the advantages of positive real controller design methods, but in addition it allows use of more realistic choices for sensors. It allows dynamic controllers, and it easily handles a nonlinear environment. A secondary advantage is the physical intuition available to help guide the control system design. Both regulator (or vibration suppression) problems and end-point control (or slewing) problems can be treated. The approach taken here exploits the work-energy rate principle in designing controllers as suggested by Oh et al.[2]

The controller design methods developed here rely on the guaranteed energy dissipation produced by a virtual mass-spring-dashpot system for their stability properties. In the large flexible spacecraft shape control problem, Canavin[3] was one of the first proponents of collocated velocity sensors and force actuators with pure velocity feedback. Such a control approach has the same guaranteed stability properties as the proposed design method, but the proposed method is more general, and does not necessarily require velocity feedback. Hyperstability and positive real controller design methods are other closely related bodies of knowledge due to Popov.[4,5] A positive real dynamic system is one that can be realized by purely passive electrical elements.[6] Benhabib et al.[7] were the first to suggest applying such concepts from network theory for shape control of large flexible spacecraft. Flashner[8] considers applications to robotics. The design of linear quadratic regulator controllers that are constrained to be positive real was studied by Sevaston and Longman.[9] Dissipative controllers for flexible spacecraft are studied by Joshi and Maghami[10] and Joshi et al.[11]

The proposed approach is much more general than typical positive real controller design. For shape control of large flexible spacecraft, positive real theory requires the use of collocated velocity sensors and force actuators (or angular rate sensors and torque actuators). The requirement of velocity sensors is very restrictive in hardware implementation. Here the same type of stability robustness properties are obtained as in positive real theory, but the use of position, acceleration, and velocity sensors is allowed, either alone or in combinations. Collocation of sensors and actuators is still required. The controller design problem is formulated in terms of a virtual mechanical system that supplies considerable intuition to guide the control system design step. The relationship and differences between the virtual passive controller and the positive real controller are studied in Ref. 12. By properly choosing the type of feedback sensors, a second-order virtual passive controller can be made to be positive real. An earlier and more complete version of the present paper appeared as Ref. 13.

This paper starts with a brief review of passive controller design for linear systems as developed in Ref. 1. The basic results for the linear case are then generalized to the nonlinear case. Insight into the passive control laws is given in the section titled Physical Interpretation. A generic configuration similar to the Shuttle's remote manipulator system (RMS) is used as an example to illustrate the proposed controller design approach.

Basic Results for Linear Systems

The virtual spring-mass-damper approach to passive controller design is first described for linear dynamic systems as in Ref. 1. Extension of the theory to general nonlinear multiple body dynamic systems is then made. A general nongyroscopic linear dynamic system can be represented as a system of second-order constant coefficient ordinary differential equations

$$M\ddot{x} + D\dot{x} + Kx = Bu \tag{1}$$

$$y = H_a\ddot{x} + H_v\dot{x} + H_d x \tag{2}$$

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where x is an $n \times 1$ coordinate vector, and M, D, and K are the symmetric mass, damping, and stiffness matrices, respectively. The $n \times p$ influence matrix B describes the actuator force distributions for the $p \times 1$ control force vector u (or generalized force). Typically, matrix M is positive definite, whereas D and K are positive definite or positive semidefinite. In the absence of rigid-body motion, K is positive definite. Equation (2) is a measurement equation having y as the $m \times 1$ measurement vector, and H_a, H_v, and H_d are the $m \times n$ acceleration, velocity, and displacement influence matrices, respectively.

Assume that the controller to be designed has a set of second-order dynamic equations and measurement equations similar to the system equations in Eqs. (1) and (2)

$$M_c\ddot{x}_c + D_c\dot{x}_c + K_c x_c = B_c u_c \tag{3}$$

$$y_c = H_{ac}\ddot{x}_c + H_{vc}\dot{x}_c + H_{dc}x_c \tag{4}$$

Here, x_c is the controller state vector of dimension n_c, and M_c, D_c, and K_c are thought of as the controller mass, damping, and stiffness matrices, respectively. These matrices are chosen to be symmetric and positive definite to make the controller equations asymptotically stable. The $n_c \times m$ influence matrix B_c describes the force distributions for the $m \times 1$ input force vector u_c. Equation (4) is the controller measurement equation having y_c as the measurement vector of length p, H_{ac} the $p \times n_c$ acceleration influence matrix, H_{vc} the $p \times n_c$ velocity influence matrix, and H_{dc} the $p \times n_c$ displacement influence matrix. The quantities M_c, D_c, K_c, B_c, H_{ac}, H_{dc}, and H_{vc} are the design parameters for the controller. Let the dynamic system in Eqs. (1) and (2) be connected to the controller system in Eqs. (3) and (4) in such a way that the output of the controller is the input to the dynamic system, and the output of the dynamic system is the input to the controller, i.e.,

$$u = y_c = H_{ac}\ddot{x}_c + H_{vc}\dot{x}_c + H_{dc}x_c \tag{5}$$

$$u_c = y = H_a\ddot{x} + H_v\dot{x} + H_d x \tag{6}$$

The overall closed-loop system then becomes

$$M_t\ddot{x}_t + D_t\dot{x}_t + K_t x_t = 0 \tag{7}$$

where

$$M_t = \begin{bmatrix} M & -BH_{ac} \\ -B_cH_a & M_c \end{bmatrix} \qquad D_t = \begin{bmatrix} D & -BH_{vc} \\ -B_cH_v & D_c \end{bmatrix}$$

$$K_t = \begin{bmatrix} K & -BH_{dc} \\ -B_cH_d & K_c \end{bmatrix} \qquad x_t = \begin{bmatrix} x \\ x_c \end{bmatrix}$$

If the system parameters M, D, K, H_a, H_d, and H_v are known, then the controller parameters M_c, D_c, K_c, H_{ac}, H_{dc}, and H_{vc} can be designed such that the closed-loop system matrices M_t, D_t, and K_t are symmetric and positive definite. This makes the closed-loop system, Eq. (7), asymptotically stable. However, it is of interest to design controllers that are insensitive to the system parameters. In the following development, this insensitivity is indeed possible provided a certain modification to the control equation is made and a certain condition on the actuator and sensor placement is satisfied. First, note the similarity in structures in the closed-loop system matrices M_t, D_t, and K_t. This similarity means that the basic design procedure for the controller parameters that appear in each of these matrices is the same. In particular, the case for designing the parameters in the stiffness matrix K_t is illustrated here. Let the actuators be located in such a way that the control influence matrix of the system equation B can be expressed by

$$B^T = Q_b H_d \tag{8}$$

Then if the control influence matrix of the control equation B_c is designed such that

$$Q_b^T H_{dc} = B_c^T \tag{9}$$

for any given matrix H_{dc}, then the resulting closed-loop stiffness matrix K_t is symmetric. To derive the conditions that would make K_t positive definite, it is adequate to consider the special case where $H_a = H_{ac} = H_v = H_{vc} = 0$. Furthermore, let the input force in Eq. (5) be modified as

$$u = y_c - Gy = H_{dc}x_c - GH_d x \tag{10}$$

where G is a gain matrix to be determined. Also, let

$$B_c = K_c\bar{B}_c \qquad \text{or} \qquad \bar{B}_c = K_c^{-1}B_c \tag{11}$$

and the gain matrix G be

$$G = H_{dc}\bar{B}_c \tag{12}$$

Then it can be shown that the closed-loop stiffness matrix in this case becomes

$$K_t = \begin{bmatrix} K + H_d^T\bar{B}_c^T K_c \bar{B}_c H_d & -H_d^T\bar{B}_c^T K_c \\ -K_c\bar{B}_c H_d & K_c \end{bmatrix} \tag{13}$$

which, in addition to being symmetric, is positive definite if the system stiffness matrix K is at least positive semidefinite. The design procedure for the other controller parameters in the closed-loop system mass and damping matrices are similar.

Generalizations to Multiple-Body Dynamic Systems

The basic results for linear systems can be generalized to the nonlinear case. This section outlines the overall methodology for extension to nonlinear systems. A detailed development of all the possibilities involved is deferred to a later paper. Certain assumptions are made in the course of the development here that must be satisfied by the physical system being controlled. Various possible controller structures are considered for different types of sensor feedback, and then a Lyapunov-like approach is taken to address the stability question. First consider multiple rigid body systems. The case of multiple flexible body systems is discussed later. Also, mechanical systems that have no external forces applied, such as robots in space, are considered.

Velocity Feedback

We start by considering controllers that use velocity feedback only. Since no position measurements are made, no attempt is made to control position. Such controllers are important by themselves for vibration suppression and are also important here as one building block in more general controller designs discussed later. Let T be the total kinetic energy of a mechanical system (linear or nonlinear) with p control actuators at p physical locations described by p generalized coordinates x_{ai}, $i = 1, 2, \ldots, p$. These generalized coordinates and their derivatives are measurable quantities such as displacements, velocities, accelerations, or their angular equivalents. If the mechanical system is holonomic and scleronomic (no explicit time dependence), a basic result of analytical mechanics relates the time derivative of the total kinetic energy $\dot{T}$ and the applied forces as[2]

$$\frac{dT}{dt} = u^T\dot{x}_a \tag{14}$$

where

$$u = (u_1 \quad u_2 \quad \cdots \quad u_p)^T$$

is the control vector with u_i $(i=1,2,\ldots,p)$ representing the generalized control force associated with the generalized coordinate x_{ai} and

$$x_a = (x_{a1} \quad x_{a2} \quad \cdots \quad x_{ap})^T$$

is a generalized coordinate vector. The physical force or torque for each actuator is a function of these generalized forces through kinematic relations. Equation (14) is referred to as the work-energy rate principle presented in Ref. 2. It indicates that the rate of change of total kinetic energy is equal to the rate of change of work produced by applied forces.

Consider T as a Lyapunov function

$$L = L(x_a, \dot{x}_a, \bar{x}, \dot{\bar{x}}) = T(x_a, \dot{x}_a, \bar{x}, \dot{\bar{x}}) \tag{15}$$

where $\bar{x}$ denotes a vector of additional generalized coordinates that need not be measurable. It is assumed that no generalized forces are generated by the controller for these extra coordinates. The time derivative of the Lyapunov function becomes

$$\dot{L} = \frac{\mathrm{d}L}{\mathrm{d}t} = \frac{\mathrm{d}T}{\mathrm{d}t} = u^T \dot{x}_a \tag{16}$$

Let the generalized control vector u be chosen such that

$$u = -D\dot{x}_a \tag{17}$$

Then it follows that

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -\dot{x}_a^T D \dot{x}_a \tag{18}$$

which is negative semidefinite in $\dot{x}_a$, $\dot{\bar{x}}$ space when D in Eq. (18) is chosen to be positive definite. Since the Lyapunov function L is positive definite in the variables $\dot{x}_a$, $\dot{\bar{x}}$, and its time rate of change $\dot{L}$ is only positive semidefinite in the same set of variables, some caution must be exercised with regard to a conclusion concerning the asymptotic stability of the overall system. If it is assumed that the mechanical structure of the system is such that $\dot{x}_a \equiv 0$ implies $\dot{\bar{x}} \equiv 0$, then it is not possible to have $\dot{L}$ identically zero and yet have the kinetic energy L fail to vanish identically. This condition is similar to controllability and observability for nonlinear systems. This assumption is satisfied provided that the manifold of the $(\dot{x}_a, \dot{\bar{x}})$ space on which the time rate of change of the Lyapunov function vanishes identically

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -\dot{x}_a^T D \dot{x}_a \equiv 0 \tag{19}$$

does not contain any arc of a trajectory, and in this case one can conclude that the trivial solution

$$\dot{x}_a = 0, \qquad \dot{\bar{x}} = 0 \tag{20}$$

is asymptotically stable. It is possible, however, to allow the trajectories to cross the manifold, as long as they do not remain in it except when at the origin. The time rate of change of the Lyapunov function considered in this case is only negative semidefinite, but the fact that no physical trajectory arcs lie in the manifold insures that over any finite time interval energy is dissipated. For our control problem, the assumption has the physical implication that it is not possible to have some coordinates still in motion without disturbing any of the controlled coordinates. Although asymptotic stability is not rigorously proved here, the energy dissipation based on the previous physical consideration implies that the equilibrium position is asymptotically stable.

Displacement Feedback

Consider displacement feedback in order to control position, or alternatively to suppress vibrational motion as in the previous section but without velocity measurements being available. Choose the Lyapunov function

$$L = L(x_a, \dot{x}_a, x_c, \dot{x}_c, \bar{x}, \dot{\bar{x}}) \tag{21}$$

where

$$L = T + ½\dot{x}_c^T M_c \dot{x}_c + ½(x_a - x_c)^T K_{c_1}(x_a - x_c) + ½x_c^T K_{c_2} x_c \tag{22}$$

where

$$x_c = (x_{c1} \quad x_{c2} \quad \cdots \quad x_{cp})^T$$

is the displacement vector corresponding to a control (virtual) mass matrix M_c, and M_c, K_{c_1}, and K_{c_2} are arbitrary positive definite matrices to be determined later. The time derivative of the Lyapunov function $\dot{L}$ becomes

$$\frac{\mathrm{d}L}{\mathrm{d}t} = u^T \dot{x}_a + \dot{x}_c^T M_c \ddot{x}_c + (\dot{x}_a - \dot{x}_c)^T K_{c_1}(x_a - x_c) + \dot{x}_c^T K_{c_2} x_c \tag{23}$$

Let u be chosen such that

$$u = -K_{c_1}(x_a - x_c) \tag{24}$$

This is a model-independent feedback law that involves only the measurable generalized coordinate vector x_a of the system and the computable displacement x_c from a controller to be determined later. The positive definite matrix K_{c_1} and the displacement vector x_c are determined to make $\dot{L}$ negative semidefinite. Substitution of Eq. (24) into Eq. (23) yields

$$\frac{\mathrm{d}L}{\mathrm{d}t} = -(x_a - x_c)^T K_{c_1} \dot{x}_a + \dot{x}_c^T M_c \ddot{x}_c + (\dot{x}_a - \dot{x}_c)^T K_{c_1}(x_a - x_c) + \dot{x}_c^T K_{c_2} x_c$$

$$\frac{\mathrm{d}L}{\mathrm{d}t} = \dot{x}_c^T M_c \ddot{x}_c - \dot{x}_c^T K_{c_1}(x_a - x_c) + \dot{x}_c^T K_{c_2} x_c$$

$$= \dot{x}_c^T \left[M_c \ddot{x}_c - K_{c_1}(x_a - x_c) + K_{c_2} x_c \right] \tag{25}$$

Let the quantity in the square bracket be

$$M_c \ddot{x}_c - K_{c_1}(x_a - x_c) + K_{c_2} x_c = -D_c \dot{x}_c \tag{26}$$

or

$$M_c \ddot{x}_c + D_c \dot{x}_c + (K_{c_1} + K_{c_2}) x_c = K_{c_1} x_a \tag{27}$$

The right-hand side of Eq. (27) involves only the measurable generalized coordinate vector x_a, which in turn determines the quantities x_c, $\dot{x}_c$ by computation. Equation (27) is a linear dynamic system represented by the controller mass matrix M_c, damping matric D_c, and stiffness matrix $(K_{c_1} + K_{c_2})$, which are all positive definite. The generalized control vector u is generated by simulating the force histories that would be applied by the chosen virtual (controller) mass-spring-dashpot system described by Eq. (27). Its physical interpretation will be shown later. Combining Eqs. (25) and (26) yields

$$\dot{L} = -\dot{x}_c^T D_c \dot{x}_c \tag{28}$$

which is negative semidefinite in the variables x_a, $\dot{x}_a$, x_c, $\dot{x}_c$, $\bar{x}$, and $\dot{\bar{x}}$. The objective of the control is to obtain

$$\lim_{t\to\infty} x_a(t) = \lim_{t\to\infty} x_c(t) = 0 \tag{29}$$

The objective can be accomplished provided the Lyapunov function considered in Eq. (22) approaches zero asymptotically. Since $\dot{L}$ is only negative semidefinite in the variables x_a,

$\dot{x}_a$, x_c, $\dot{x}_c$, $\bar{x}$, and $\dot{\bar{x}}$, in general it is possible to have $\dot{L}$ vanish, yet the Lyapunov function L itself does not. Assuming that the system is configured such that it is not possible to have some coordinates still in motion without disturbing any of the controlled coordinates, i.e., $\dot{x}_a \equiv 0$ implies $\dot{\bar{x}} \equiv 0$, then in the limit L must approach zero as t tends to infinity. This can be seen by considering the case where $\dot{x}_c \equiv 0$. Note that $\dot{x}_c \equiv 0$ implies $\dot{x}_a \equiv 0$. Therefore, the first two terms in the Lyapunov function, Eq. (22), vanish. Furthermore, in the absence of external forces other than control forces, assume that $\dot{x}_c \equiv 0$ implies $x_c \equiv 0$, which from Eq. (27) yields $x_a \equiv 0$ provided K_{c_1} is invertible. This makes the last two terms in the Lyapunov function vanish as well.

Computation of the quantity x_c requires measurement of the vector x_a as shown in Eq. (27). Therefore, when velocity sensors are not available, so that the control law cannot depend on measuring $\dot{x}_a$ as in the previous section, it is still possible to make $\dot{L}$ negative semidefinite using the feedback law given in Eq. (24) and accomplish vibration suppression. The positive definite matrices M_c, D_c, K_{c_1}, and K_{c_2} are arbitrary and can be chosen to meet performance requirements.

Displacement and Acceleration Feedback

Feedback laws using acceleration measurements can be derived similarly. This is an important case for vibration suppression in structures since acceleration measurements are much more easily made than velocity measurements. Consider a Lyapunov function of the form

$$L = T + \tfrac{1}{2}(\dot{x}_a+\dot{x}_c)^T M_c(\dot{x}_a+\dot{x}_c) + \tfrac{1}{2}x_c^T K_{c_1} x_c + \tfrac{1}{2}(x_a+x_c)^T K_{c_2}(x_a+x_c) \tag{30}$$

The time derivative of L then becomes

$$\frac{dL}{dt} = u^T\dot{x}_a + (\dot{x}_a+\dot{x}_c)^T M_c(\ddot{x}_a+\ddot{x}_c) + \dot{x}_c^T K_{c_1} x_c + (\dot{x}_a+\dot{x}_c)^T K_{c_2}(x_a+x_c) \tag{31}$$

Let the control input be chosen such that

$$u = -M_c(\ddot{x}_a+\ddot{x}_c) - K_{c_2}(x_a+x_c) \tag{32}$$

Then the time rate of change of the Lyapunov function becomes

$$\frac{dL}{dt} = -(\ddot{x}_a+\ddot{x}_c)^T M_c\dot{x}_a - (x_a+x_c)^T K_{c_2}\dot{x}_a + (\dot{x}_a+\dot{x}_c)^T M_c(\ddot{x}_a+\ddot{x}_c) + x_c^T K_c\dot{x}_c + (\dot{x}_a+\dot{x}_c)^T K_{c_2}(x_a+x_c)$$

$$\frac{dL}{dt} = \dot{x}_c^T M_c(\ddot{x}_a+\ddot{x}_c) + x_c^T K_{c_1}\dot{x}_c + \dot{x}_c^T K_{c_2}(x_a+x_c) - (x_a+x_c)^T K_{c_2}\dot{x}_a + \dot{x}_a^T K_{c_2}(x_a+x_c)$$

$$\frac{dL}{dt} = \dot{x}_c^T\left[K_{c_1}x_c + M_c(\ddot{x}_a+\ddot{x}_c) + K_{c_2}(x_a+x_c)\right] \tag{33}$$

If the quantity in the square bracket in the previous equation is set to be

$$K_{c_1}x_c + M_c(\ddot{x}_a+\ddot{x}_c) + K_{c_2}(x_a+x_c) = -D_c\dot{x}_c \tag{34}$$

or

$$M_c\ddot{x}_c + D_c\dot{x}_c + (K_{c_1}+K_{c_2})x_c = -M_c\ddot{x}_a - K_{c_2}x_a \tag{35}$$

where D_c is an arbitrary positive definite matrix, then Eq. (33) becomes

$$\frac{dL}{dt} = -\dot{x}_c^T D_c\dot{x}_c \tag{36}$$

which is negative semidefinite in the variables x_a, $\dot{x}_a$, x_c, $\dot{x}_c$, $\bar{x}$, and $\dot{\bar{x}}$. Note that even in the absence of velocity measurements, the feedback control law given in Eq. (32), which is based on acceleration and displacement measurements, is sufficient to make $\dot{L}$ negative semidefinite. Again, assuming that $\dot{x}_a \equiv 0$ implies $\dot{\bar{x}} \equiv 0$, it can be concluded that L approaches zero as t tends to infinity. The line of argument follows closely the previous development in the case of displacement feedback and hence is omitted.

Displacement, Velocity, and Acceleration Feedback

The general case of displacement, velocity, and acceleration feedback can be easily derived by considering a Lyapunov function of the form

$$L = T + L_1 + L_2 \tag{37}$$

where

$$L_1 = \tfrac{1}{2}\dot{x}_{cd}^T M_{cd}\dot{x}_{cd} + \tfrac{1}{2}(x_a-x_{cd})^T K_{cd_1}(x_a-x_{cd}) + \tfrac{1}{2}x_{cd}^T K_{cd_2}x_{cd}$$

$$L_2 = \tfrac{1}{2}(\dot{x}_a+\dot{x}_{ca})^T M_{ca}(\dot{x}_a+\dot{x}_{ca}) + \tfrac{1}{2}x_{ca}^T K_{ca_1}x_{ca} + \tfrac{1}{2}(x_a+x_{ca})^T K_{ca_2}(x_a+x_{ca})$$

The control law is then of the form

$$u = -D\dot{x}_a - K_{cd_1}(x_a-x_{cd}) - M_{ca}(\ddot{x}_a+\ddot{x}_{ca}) - K_{ca_2}(x_a+x_{ca}) \tag{38}$$

where the vectors x_{cd} and x_{ca} are determined from

$$M_{cd}\ddot{x}_{cd} + D_{cd}\dot{x}_{cd} + (K_{cd_1}+K_{cd_2})x_{cd} = -K_{cd_1}x_a$$
$$M_{ca}\ddot{x}_{ca} + D_{ca}\dot{x}_{ca} + (K_{ca_1}+K_{ca_2})x_{ca} = -M_{ca}\ddot{x}_a - K_{ca_2}x_a \tag{39}$$

The coefficient matrices as well as K_{cd_1} and K_{ca_2} in Eq. (39) are all required to be positive definite. The time rate of change of the Lyapunov function then becomes

$$\dot{L} = -\dot{x}_a^T D\dot{x}_a - \dot{x}_{cd}^T D_{cd}\dot{x}_{cd} - \dot{x}_{ca}^T D_{ca}\dot{x}_{ca} \tag{40}$$

which is negative semidefinite. The proof of stability parallels the previous case.

Generalization to Multiple Flexible Body Systems with Viscous Friction

Including flexibility into the bodies of the system introduces a potential energy of deformation. In many cases, such as robots with controllers at each joint, this potential energy is special in the sense that it does not alter the equilibrium position of the system including the virtual controller. Since the potential energy function does not change the equilibrium position, or introduce new equilibria, there is no need to solve for the equilibrium before studying stability. To include such a potential energy in the analysis, one simply notes that the time rate of change of the kinetic energy in Eq. (14) becomes the time rate of change of the total energy[2] so that

$$\frac{d}{dt}(T+V) = -2F_d + u^T\dot{x}_a \tag{41}$$

where F_d is the quadratic Rayleigh's dissipation function included here to handle any viscous damping in the system. All the theory developed earlier applies again. One simply replaces the kinetic energy T by the total mechanical energy $E = T + V$ of the system in the related expressions.

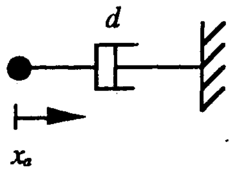

Fig. 1 Velocity feedback.

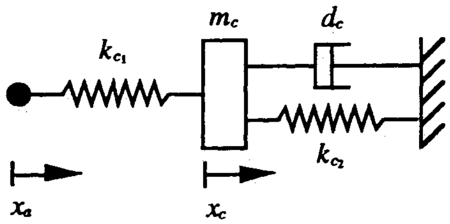

Fig. 2 Displacement feedback.

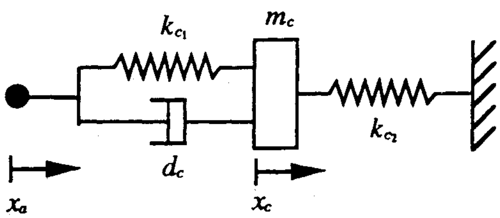

Fig. 3 Displacement and acceleration feedback.

Physical Interpretation

Insight into the control laws developed can be gained through the following physical interpretation in terms of mechanical analogs. These analogs motivated the previous mathematical formulations. For simplicity, consider only the single input and single output case. The matrices M_c, D_c, and K_c reduce to scalar quantities and are denoted by the lower case letters m_c, d_c, and k_c.

Velocity Feedback

The controller is given by Eq. (17). The control force applied is equivalent to the force transmitted to the system at the actuator location by a dashpot with a damping coefficient d. This result is shown in Fig. 1.

If x_a is an angular displacement, then the control law corresponds to an angular dashpot.

Displacement Feedback

The control law is given in Eq. (24). Examination of Eqs. (24) and (27) shows that the control force is equivalent to that provided by a mass-spring-dashpot system at the actuator location. This result is shown in Fig. 2.

In this case, the quantity x_a is the measured inertial position of the system at the actuator location, whereas the quantity x_c is defined as the inertial position of the virtual mass m_c. The springs are considered as ideal springs, and their unstretched lengths can be considered to be zero.

Displacement and Acceleration Feedback

The control law is given in Eqs. (32) and (35). Examination of the controller equations verifies that the control force is equivalent to a mass-spring-dashpot system at the actuator location, with one important distinguishing feature. The position of the controller (virtual) mass x_c shown in Fig. 3 is now defined relative to the measured inertial position x_a of the system at the actuator location whereas in the earlier displacement feedback case, it is defined as an inertial position. The free lengths of the springs are again taken as zero.

Remarks

Remark 1: Certain simplifications of the controllers developed here are worth noting. In both the displacement feedback and displacement and acceleration feedback cases, the inclusion of the spring characterized by its coefficient k_{c_2} is necessary for position control but not vital for vibration suppression. This special feature allows one to design robust vibration suppression controllers with accelerometer feedback. In particular, setting $k_{c_2}=0$ in the displacement and acceleration feedback law, Eq. (32), results in a feedback law based on acceleration measurements alone. The corresponding diagram for this case is the same as the one given in Fig. 3 with the spring connected to the ground removed.

Remark 2: In the absence of direct velocity measurements, it is possible to mimic direct velocity feedback by displacement measurements alone. Consider the case of displacement feedback as shown in Fig. 2. By letting $k_{c_2}=0$, and choosing k_{c_1} to be large and m_c to be small, the behavior of the resulting mass-spring-dashpot system approximates that of a single dashpot. This is equivalent to putting a controller zero near the origin but not at the origin.

Remark 3: It is also possible to approximate direct velocity feedback from displacement and acceleration measurements. Refer to Fig. 4. If k_{c_1} and m_c are chosen to be small and k_{c_2} to be large, then the behavior of the resultant mass-spring-dashpot system approximates that of a single dashpot. In the absence of displacement measurements ($k_{c_2}=0$), the same effect can be achieved by choosing k_{c_1} to be small but m_c to be large.

Remark 4: To implement the controller in Fig. 2 or 3, it is necessary to select the correct initial conditions for the controller coordinate x_c to be used. There is a subtle difference between the vibration suppression problem and the end-point control problem by this approach, even though they may be viewed as the same conceptually. For vibration suppression, the controller consists of second-order dynamic systems whose equilibrium position of the controller mass is consistent with the desired equilibrium configuration of the structure. Assuming that the structure is initially at rest, the initial condition for the controller coordinate is then $x_c=\dot{x}_c=0$. For end-point positioning control, however, the initial conditions for the controller coordinate must be chosen such that in the desired final configuration of the manipulator, the virtual spring and dashpot are in their equilibrium and unstretched position. In the end-point control problem, the induced vibrations are also suppressed by this controller design.

Remark 5: When the controllers are interpreted in physical terms, recall that for the case of displacement feedback, the position vector of the controller (virtual) mass x_c in the Lyapunov function, Eq. (22), is defined with respect to an inertial reference. However, for displacement and acceleration feedback, Eq. (30), it is defined relative to the actuator location. When the control mass location is expressed as an inertial

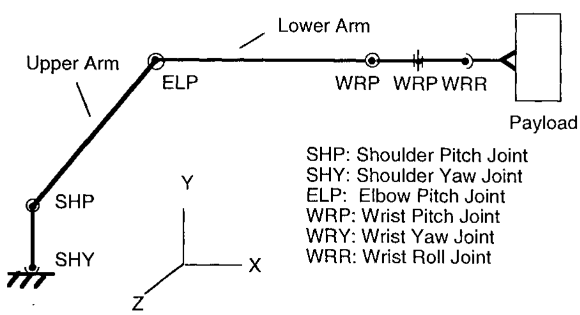

Fig. 4 Six-degree-of-freedom robot.

position, the Lyapunov function in the second case becomes identical to that in the first case. The difference is in the formulation of the control law. By expressing it in terms of the relative position of the controller mass, the scheme can be implemented by acceleration measurements without the need for velocity measurements.

Numerical Examples

The robot system shown in Fig. 4 consists of six one-degree-of-freedom joints, namely, shoulder pitch, yaw, elbow pitch, wrist yaw, pitch, and roll. This robot model has the same kinematic relationships and mass properties as the Shuttle's RMS. The shoulder and elbow joints are connected by a 6.4-m-long lightweight carbon composite boom. This boom is designated the upper arm boom. The lower arm boom, connecting the elbow joint to the wrist joints, is approximately 7 m long. The shoulder and elbow joints provide three translational degrees of freedom, and the wrist joints provide three additional rotational degrees of freedom of the end effector. The geometric and material properties of the flexible arms are given in Ref. 13.

A simple maneuver to move the end effector from its current position to a new position is simulated. Initially, the robot system is in the position shown in Fig. 4, and the position vector of the end effector is (15.3162, 0.3048, 0.0) given in terms of its x, y, and z components. The desired new position of the end effector relative to the robot base is (10.3162, 5.3048, 5.0). Assume that all of the wrist joints are locked during the maneuvering and control torques are applied only to each movable joint, i.e., the shoulder yaw, pitch, and elbow pitch joints. Since the system is driven by torques at the joints, the required change in each joint angle is precalculated to be −0.45131, −0.3111, and 1.2106 rad for the shoulder yaw, pitch, and elbow pitch joints, respectively. In this case, a second-order virtual rotational spring-mass damper system is located at each joint to be controlled. The controller system then has virtual moments of inertia instead of virtual masses, torsional springs and dashpots instead of linear springs and dashpots. The initial conditions of the control system are chosen such that in the final desired configuration of the manipulator, the virtual springs assume their free-length forms.

In the simulations, the wrist links are modeled as rigid bodies since they are fairly short relative to the lengths of both arms. The upper and lower arms are modeled as flexible bodies. Only the first three cantilever modes are used to represent the flexibility of the links. Three different controllers, distinguished by the measured signals, are used to illustrate the use of the controller design methods in this paper.

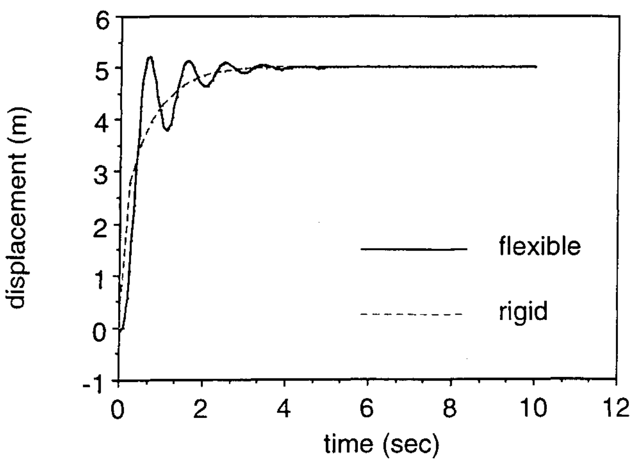

Fig. 5 Time histories of the z position of the end effector, case 1.

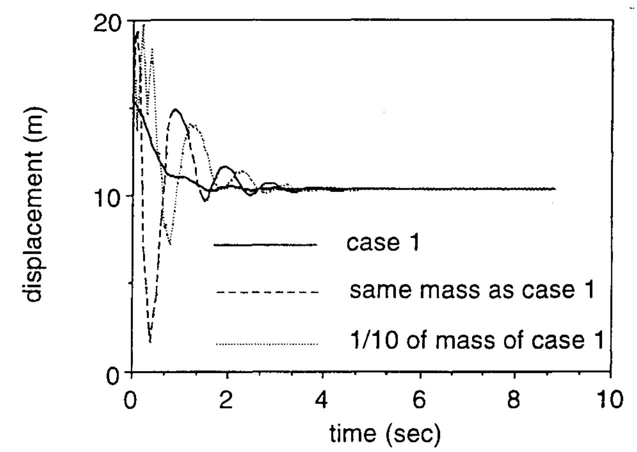

Fig. 6 Time histories of the x position of the end effector, case 2.

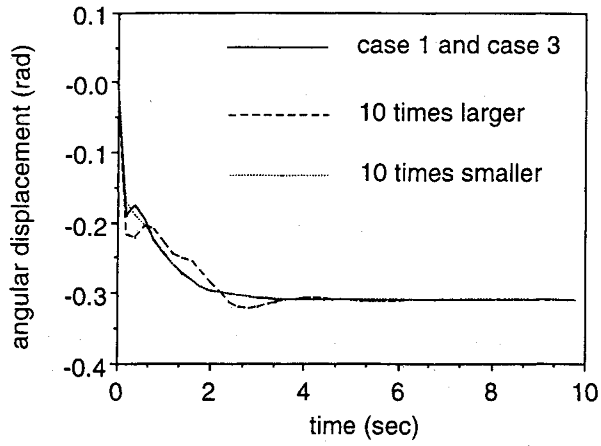

Fig. 7 Time histories of the displacement of the shoulder pitch for different values of control mass constant, case 3.

Case 1: Displacement and Velocity Feedback

Case 1 uses angular displacement and velocities of joints for feedback. The controllers for each joint are rotational versions of the diagram in Figs. 1 and 2. The parameters of the controller are given in Ref. 13. No attempt has been made to optimize the parameters chosen. Figure 5 shows the z-position time history of the end effector in both flexible- and rigid-body simulations. The results show that the vibration is actively suppressed.

Case 2: Displacement Feedback

One advantage of the virtual system controller design approach developed here is that velocity feedback is not required for vibration control or stability robustness. The stability robustness, however, need not guarantee good damping rates. To attempt to produce responses like those in case 1, but this time using displacement measurement feedback alone, the order of the controller is increased to emulate velocity feedback. This change is accomplished by replacing the velocity feedback portion of case 1 with a rotational virtual system corresponding to Fig. 2 and using a large k_{c_1} and a small m_c. Theoretically, when both the k_{c_1} and m_c of the middle connection go to their limits, the control system in case 2 is equivalent to the control system in case 1. Numerically, the values of these constants are constrained by the introduction of high frequencies and real-time numerical integration difficulties.

In the simulation, the value of k_{c_1} of the second virtual system is taken to be 100 times stiffer than the k_{c_1} of the first virtual system. Figure 6 shows the simulation results with two different values of m_c for the second virtual system and shows that this approach will require rather extreme mass and stiffness values for good damping performance.

Case 3: Displacement and Displacement-Acceleration Feedback

Another way to emulate the velocity signal is by using displacement-acceleration feedback. By replacing the velocity feedback in case 1 with the displacement-acceleration feedback of Fig. 3 and using large k_{c_2} and small m_c in this virtual system, the controller works similarly to the controller used in case 1. Theoretically, when the k_{c_2} and m_c of the acceleration connection go to their limits, the control system in case 3 is equivalent to the control system in case 1. The controller in case 3 is numerically better conditioned than that in case 2, since the velocity signal is obtained in the limit through integration of acceleration rather than differentiation of displacement.

In the simulation, the value of k_{c_2} of the second virtual system is taken to be 10 times stiffer than the k_{c_2} of the first virtual system. The results obtained are identical to within plotting accuracy to the results obtained in case 1, and are therefore not shown here. Since acceleration feedback is often more realistic than velocity feedback, this example shows that the control design of case 3 can be important in practical applications.

So far, the controller parameters have not been optimized. To get some understanding of how the parameters of the virtual system affect performance, several cases are simulated with adjusted m_c, k_{c_2}, and d_c of the first virtual control system in case 3. Each of the parameters is changed to 10 times larger and 10 times smaller than its original value. Figure 7 gives results obtained when varying the control mass m_c. In this particular example, the smaller the controller mass, the better the response.

Conclusions

In this paper, a general control design methodology has been outlined for large angle position control and vibration suppression in multiple flexible-body dynamic systems. The method guarantees stability of the controlled system, whether linear or nonlinear. The approach is model-independent in the sense that knowledge of the system dynamics is not required in the design process. Hence, it is robust with respect to parameter variations. Unlike positive real controller design, which allows velocity measurements only, the current development can use velocity, position, and acceleration measurements or any combination and still guarantee stability of the closed-loop system. In addition it allows use of dynamic feedback controllers with additional opportunity for tuning to obtain good system performance. One special case of the methodology demonstrates how one can obtain the desirable properties of velocity feedback using acceleration feedback, which is often easier to instrument in practice. The design has intuitive appeal in terms of its physical interpretation that aids the control design process.

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